Name: SOLUTIONS

GCSE (1 - 9)

Area of any Triangle

Buy me a coffee

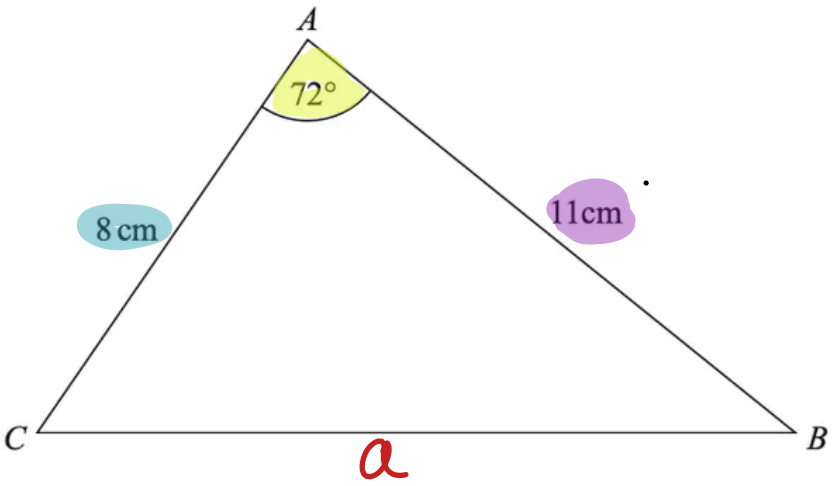
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These questions are compiled from *released* Pearson Edexcel GCSE maths exam papers
They contain exam questions from up until the ***November 2021*** exam series
More recent exam papers do exist but are withheld for mock exams
As new exam papers are released these worksheets will gradually be updated
The ***most current*** version of these worksheets will always be available at: kennethstafford.co.uk
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Question	Where the question appeared
1	Edexcel, GCSE, Autumn 2021, Paper 2H, Q15
2	Edexcel, GCSE, Autumn 2021, Paper 3H, Q23
3	Edexcel, GCSE, Autumn 2020, Paper 3H, Q19
4	Edexcel, GCSE, Autumn 2019, Paper 2H, Q23
5	Edexcel, GCSE, Summer 2019, Paper 3H, Q14
6	Edexcel, GCSE, Autumn 2018, Paper 2H, Q13
7	Edexcel, GCSE, Autumn 2017, Paper 3H, Q17
8	Edexcel, GCSE, Summer 2017, Paper 2H, Q17
9	Edexcel, GCSE, Summer 2017, Paper 3H, Q15

1

Here is triangle ABC.



(a) Find the length of BC.
Give your answer correct to 3 significant figures.

$a^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \times \cos(72)$

$a^2 = 130.613$

$a = \sqrt{130.613} = 11.4286$

11.4 ..... cm

(3)

(b) Find the area of triangle ABC.
Give your answer correct to 3 significant figures.

$\frac{1}{2} \times 8 \times 11 \times \sin(72)$

$= 41.846$

41.8 ..... cm^2

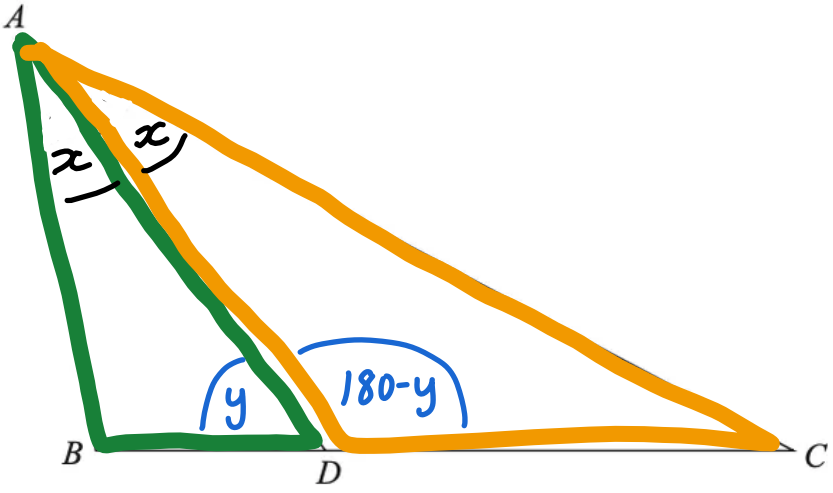
(2)

(Total for Question 15 is 5 marks)

[Edexcel, GCSE, Autumn 2021, Paper 2H, Q15]

2

ABC is a triangle.



D is the point on BC such that angle BAD = angle $DAC = x°$

Prove that $\frac{AB}{BD} = \frac{AC}{DC}$

$\frac{BD}{\sin x} = \frac{AB}{\sin y}$

$\frac{DC}{\sin x} = \frac{AC}{\sin(180-y)}$

$\frac{\sin y}{\sin x} = \frac{AB}{BD}$

$\frac{\sin(180-y)}{\sin x} = \frac{AC}{DC}$

$\sin(180-y) = \sin y$

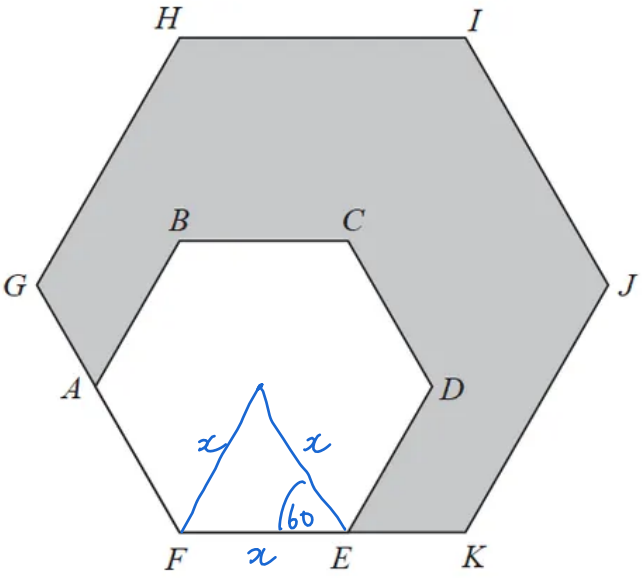
So $\frac{\sin y}{\sin x} = \frac{AC}{DC}$

So $\frac{AC}{DC} = \frac{AB}{BD}$

(Total for Question 23 is 4 marks)

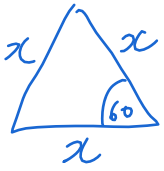
[Edexcel, GCSE, Autumn 2021, Paper 3H, Q23]

3



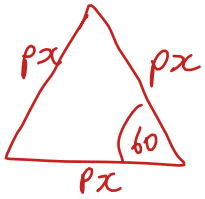
ABCDEF is a regular hexagon with sides of length x.
This hexagon is enlarged, centre F, by scale factor p to give hexagon *FGHIJK*.

Show that the area of the shaded region in the diagram is given by $\frac{3\sqrt{3}}{2}(p^2 - 1)x^2$



Area $= \frac{1}{2} \times (x)(x) \times \sin(60) = \frac{\sqrt{3}x^2}{4}$

Area of hexagon $= 6 \times \frac{\sqrt{3}x^2}{4} = \frac{3\sqrt{3}x^2}{2}$



Area $= \frac{1}{2}(px)(px) \times \sin(60) = \frac{\sqrt{3}p^2x^2}{4}$

Area of hexagon $= 6 \times \frac{\sqrt{3}p^2x^2}{4} = \frac{3\sqrt{3}p^2x^2}{2}$

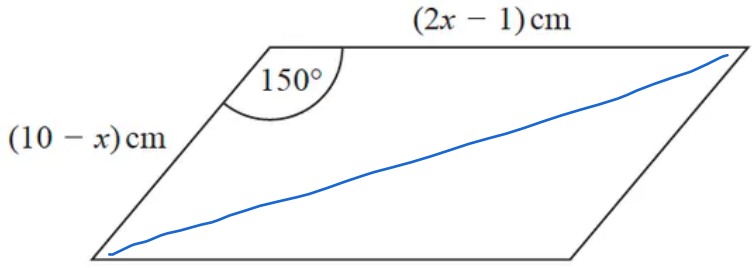
Shaded Area $= \frac{3\sqrt{3}p^2x^2}{2} - \frac{3\sqrt{3}x^2}{2} = \frac{3\sqrt{3}p^2x^2 - 3\sqrt{3}x^2}{2} = \frac{3\sqrt{3}}{2}(p^2 - 1)x^2$

(Total for Question 19 is 4 marks)

[Edexcel, GCSE, Autumn 2020, Paper 3H, Q19]

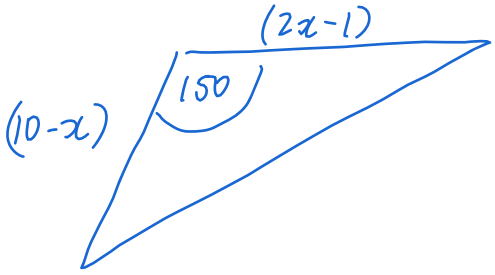
4

The diagram shows a parallelogram.



The area of the parallelogram is greater than 15 cm²

(a) Show that $2x^2 - 21x + 40 < 0$



$\frac{1}{2}(10-x)(2x-1) \times \sin(150)$

$\frac{1}{2}(20x - 10 - 2x^2 + x) \times \frac{1}{2}$

Triangle $= \frac{1}{4}(-2x^2 + 21x - 10)$

Parallelogram $= 2 \times \frac{1}{4}(-2x^2 + 21x - 10) > 15$

$\frac{1}{2}(-2x^2 + 21x - 10) > 15$

$-2x^2 + 21x - 10 > 30$

$2x^2 - 21x + 10 < -30$

$2x^2 - 21x + 40 < 0$ (3)

(b) Find the range of possible values of x.

$2x^2 - 21x + 40 < 0$

$(2x - 5)(x - 8) < 0$

$\frac{5}{2} < x < 8$

$2.5 < x < 8$

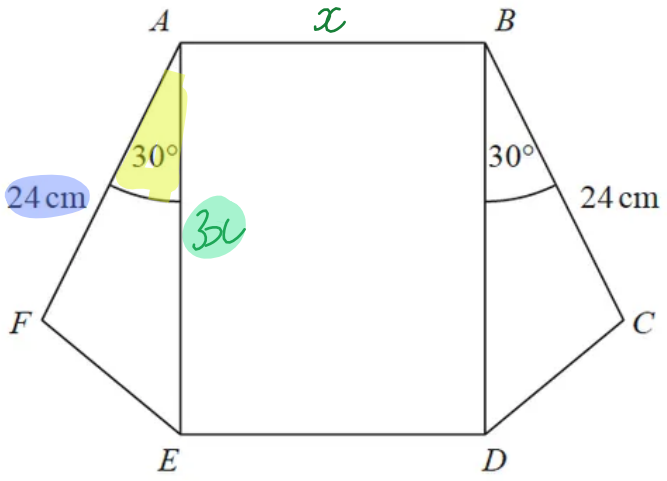
(3)

(Total for Question 23 is 6 marks)

[Edexcel, GCSE, Autumn 2019, Paper 2H, Q23]

5

The diagram shows a rectangle, $ABDE$, and two congruent triangles, AFE and BCD.



area of rectangle $ABDE$ = area of triangle AFE + area of triangle BCD

$AB : AE = 1 : 3$

$x : 3x$

Work out the length of AE.

area of triangle

$2 \times \left[\frac{1}{2} \times 24 \times 3x \times \sin(30)\right]$ = area of rectangle $x(3x)$

$72x \times \sin(30) = 3x^2$

$72x \times \frac{1}{2} = 3x^2$

$36x = 3x^2$

$12x = x^2$

$x^2 - 12x = 0$

$x(x - 12) = 0$

$x = 12$

$12 \times 3 = 36$

..... cm

(Total for Question 14 is 4 marks)

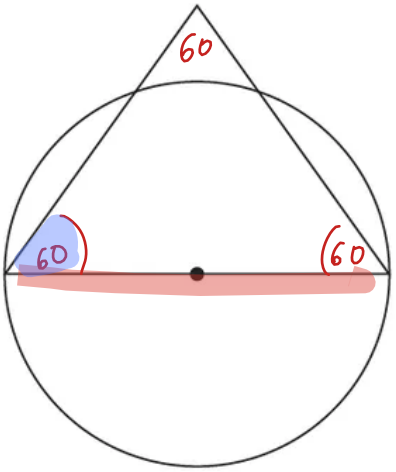
[Edexcel, GCSE, Summer 2019, Paper 3H, Q14]

6

The diagram shows a circle and an equilateral triangle.

One side of the equilateral triangle is a diameter of the circle.
The circle has a circumference of 44 cm.

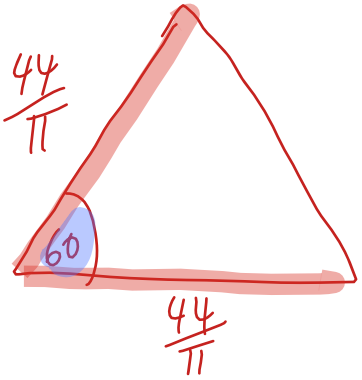
Work out the area of the triangle.
Give your answer correct to 3 significant figures.



$d = \frac{44}{\pi}$

$C = \pi d$

$d = \frac{C}{\pi}$



$\text{Area} = \frac{1}{2} \times \frac{44}{\pi} \times \frac{44}{\pi} \times \sin(60)$

$= 84.938$

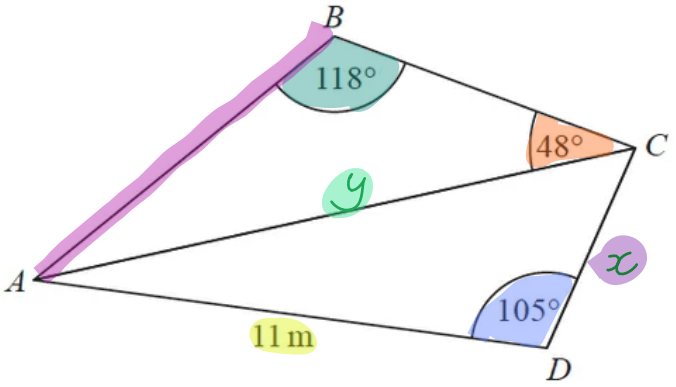
84.9 ..... cm^2

(Total for Question 13 is 3 marks)

[Edexcel, GCSE, Autumn 2018, Paper 2H, Q13]

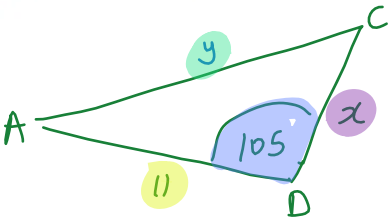
7

ABC and *ADC* are triangles.



The area of triangle *ADC* is 56 m²

Work out the length of *AB*.
Give your answer correct to 1 decimal place.



$\frac{1}{2} \times 11 \times x \times \sin(105) = 56$

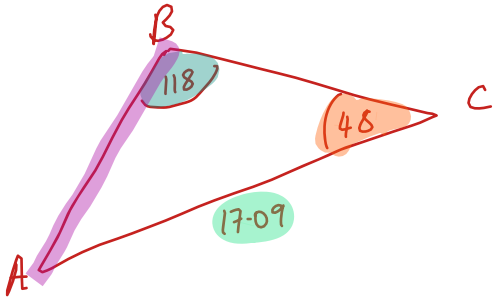
$x \times 11\sin(105) = 112$

$x = \frac{112}{11\sin(105)} = 10.54$

$y^2 = 11^2 + 10.54^2 - 2 \times 11 \times 10.54 \cos(105)$

$y^2 = 292.106$

$y = 17.09$



$\frac{AB}{\sin(48)} = \frac{17.09}{\sin(118)}$

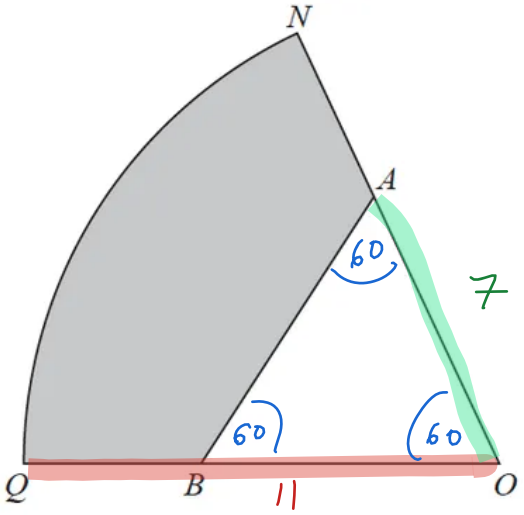
$AB = \frac{17.09\sin(48)}{\sin(118)} = 14.38$

14.4 ..... m

(Total for Question 17 is 5 marks)

[Edexcel, GCSE, Autumn 2017, Paper 3H, Q17]

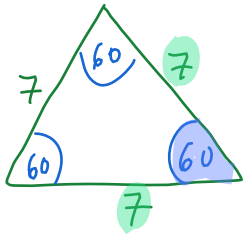
8



ONQ is a sector of a circle with centre *O* and radius 11 cm.

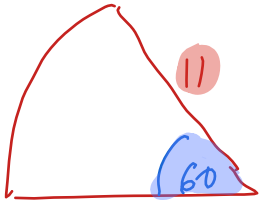
A is the point on *ON* and *B* is the point on *OQ* such that *AOB* is an equilateral triangle of side 7 cm.

Calculate the area of the shaded region as a percentage of the area of the sector *ONQ*.
Give your answer correct to 1 decimal place.



Area $= \frac{1}{2} \times 7 \times 7 \times \sin(60)$

$= 22.218 \text{ cm}^2$



Area $= \frac{60}{360} \times \pi \times 11^2 = 63.355 \text{ cm}^2$

Shaded Area $= 63.355 - 22.218 = 41.137$

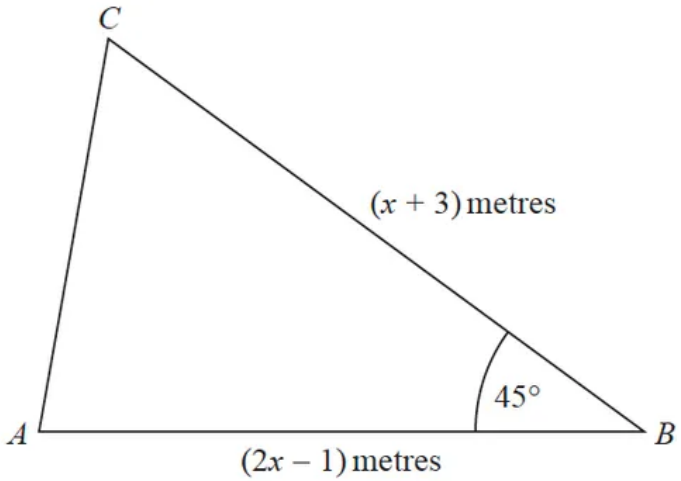
Percentage of sector $= \frac{41.137}{63.355} \times 100 = 64.93\%$

64.9 %

(Total for Question 17 is 5 marks)

[Edexcel, GCSE, Summer 2017, Paper 2H, Q17]

9



The area of triangle ABC is $6\sqrt{2}$ m^2.

Calculate the value of x.
Give your answer correct to 3 significant figures.

$\frac{1}{2}(2x-1)(x+3) \times \sin(45) = 6\sqrt{2}$

$(2x^2+5x-3) \times \frac{\sqrt{2}}{2} = 12\sqrt{2}$

$2x^2+5x-3 = \frac{12\sqrt{2}}{1} \times \frac{2}{\sqrt{2}}$

$2x^2+5x-3 = 24$

$2x^2+5x-27 = 0$

$x = \frac{-5 \pm \sqrt{5^2-4\times2\times-27}}{2\times2}$

$x = 2.631$ or $x = -5.131$ can't be negative

2·63

(Total for Question 15 is 5 marks)

[Edexcel, GCSE, Summer 2017, Paper 3H, Q15]