Name: SOLUTIONS

GCSE (1 - 9)

Circle Theorems

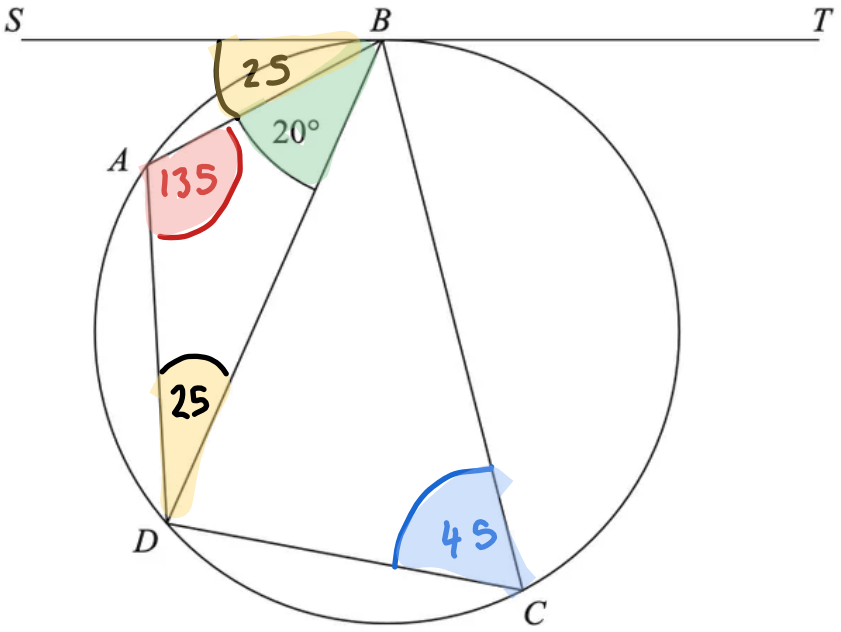
Buy me a coffee

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These questions are compiled from *released* Pearson Edexcel GCSE maths exam papers
They contain exam questions from up until the ***Autumn 2021*** exam series
More recent exam papers do exist but are withheld for mock exams
As new exam papers are released these worksheets will gradually be updated
The ***most current*** version of these worksheets will always be available at: kennethstafford.co.uk
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Question	Paper where the question appeared
1	Edexcel, GCSE, Autumn 2021, Paper 2H, Q14
2	Edexcel, GCSE, Autumn 2020, Paper 3H, Q14
3	Edexcel, GCSE, Autumn 2019, Paper 1H, Q22
4	Edexcel, GCSE, Autumn 2019, Paper 2H, Q17
5	Edexcel, GCSE, Summer 2019, Paper 2H, Q18
6	Edexcel, GCSE, Autumn 2018, Paper 1H, Q12
7	Edexcel, GCSE, Autumn 2018, Paper 2H, Q21
8	Edexcel, GCSE, Summer 2018, Paper 1H, Q11
9	Edexcel, GCSE, Summer 2018, Paper 2H, Q13
10	Edexcel, GCSE, Autumn 2017, Paper 3H, Q20
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12	Edexcel, GCSE, Summer 2017, Paper 3H, Q18

1



A, B, C and D are four points on a circle.
SBT is a tangent to the circle.
Angle $ABD = 20°$

the size of angle BAD : the size of angle $BCD = 3 : 1$

Find the size of angle SBA.
Give a reason for each stage of your working.

$3x : x = 180$
$135 : 45 = 180$

BAD = 135 (opposite angles in a cyclic quadrilateral sum to 180°)

BCD = 45°

BDA = 25° (angles in a triangle sum to 180°)

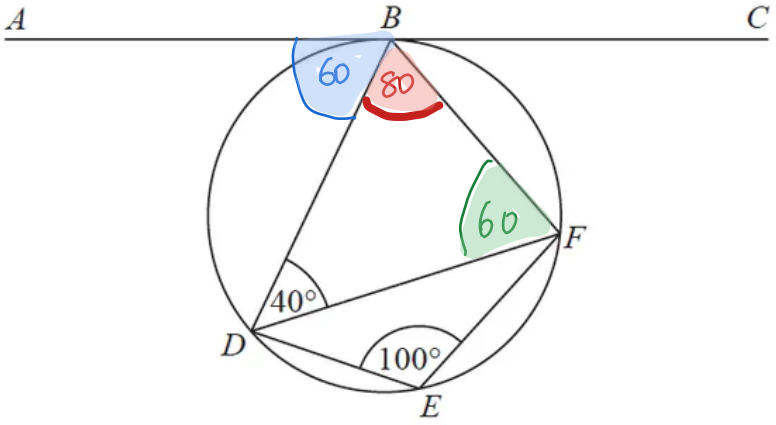
SBA = BDA = 25° angle between a tangent and a chord is equal to the angle in the alternate segment.

°

(Total for Question 14 is 4 marks)

[Edexcel, GCSE, Autumn 2021, Paper 2H, Q14]

2



Points B, D, E and F lie on a circle.
ABC is the tangent to the circle at B.

Find the size of angle ABD.
You must give a reason for each stage of your working.

$DBF = 80°$ Opposite angles in a cyclic quadrilateral sum to 180.

$DFB = 60°$ Angles in a triangle sum to 180.

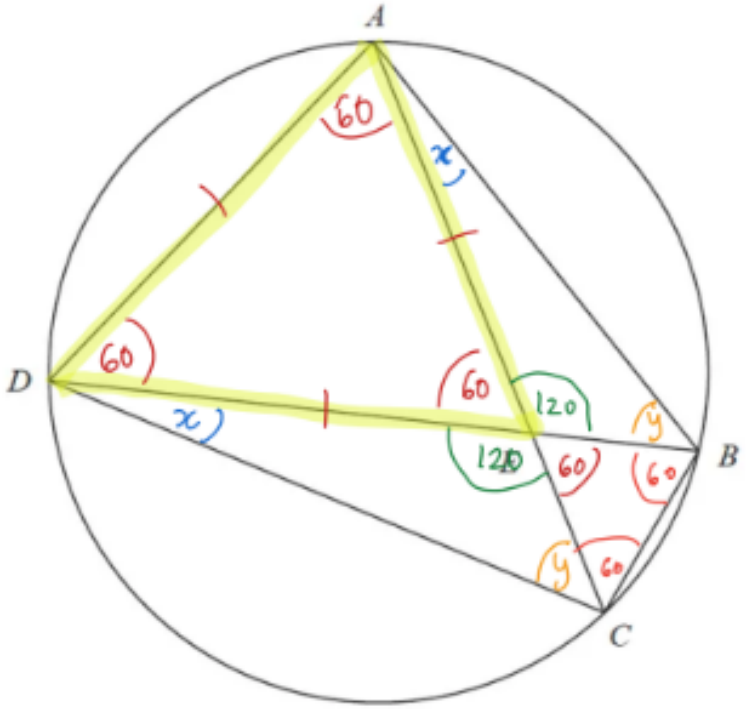
$ABD = 60°$ ABD = BFD. Angle between a tangent and a curve is the same as the angle in the alternative segment

(Total for Question 14 is 4 marks)

[Edexcel, GCSE, Autumn 2020, Paper 3H, Q14]

3

A, B, C and D are four points on a circle.



AEC and DEB are straight lines.

Triangle AED is an equilateral triangle.

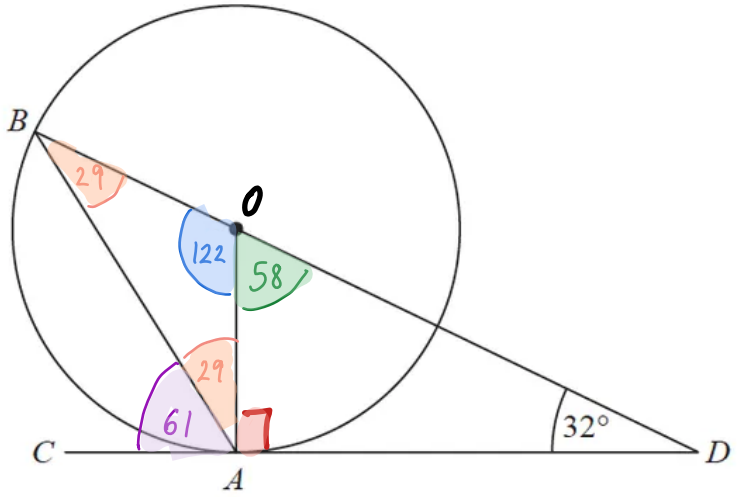
Prove that triangle ABC is congruent to triangle DCB.

for one correct pair of equal angles with correct reason from:
angle ACB = angle ADB, (angles in the same segment are equal)
angle DBC = angle DAC, (angles in the same segment are equal)
angle ABD = angle ACD, (angles in the same segment are equal)
or for recognising all angles of 60 in triangle AED and in triangle CEB)
for one identity, with reason(s), from the following list of alternatives:
Alternatives:
a complete method to show that angle ACB = angle DBC (= 60),
or
BC being common to both triangles
or
DB = DE+EB = AE+EC = AC (sides of an equilateral triangle are equal)
or
angle ABC = 60 + angle ABD = 60 + angle ACD = angle DCB (angles in the same segment are equal)
or
angle BDC = angle CAB (angles in the same segment are equal)
Underlined words need to be shown; reasons need to be linked to their statement(s)
Pairs of equal angles may be just shown on the diagram
C1 for a second identity, with reason(s), from the alternatives above
C1 for concluding the proof with a third identity, with reason(s), from the alternatives above, together with the condition for congruency, ASA or SAS or AAS

(Total for Question 22 is 4 marks)

[Edexcel, GCSE, Autumn 2019, Paper 1H, Q22]

4



A and B are points on a circle with centre O.
CAD is the tangent to the circle at A.
BOD is a straight line.

Angle $ODA = 32°$

Work out the size of angle CAB.
You must show all your working.

OAD = 90°
AOD = 58 (angles in a triangle)
BOA = 122 (angles on a straight line)
OBA = OAB = 29 (base angles in an isosceles are equal).

BAC = 61 (90 - 29)

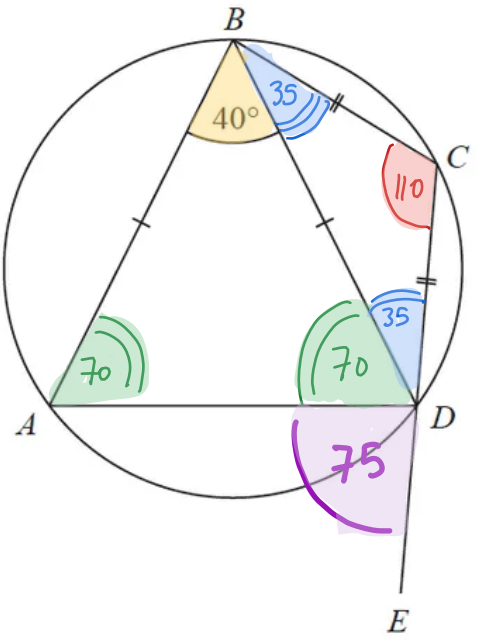
61 °

(Total for Question 17 is 3 marks)

[Edexcel, GCSE, Autumn 2019, Paper 2H, Q17]

5

The points A, B, C and D lie on a circle.
CDE is a straight line.



$BA = BD$
$CB = CD$
Angle $ABD = 40°$

Work out the size of angle ADE.
You must give a reason for each stage of your working.

BAD = BDA = 70. Base angles in an isosceles are equal

BCD = 110. Opposite angles in acyclical quadrilateral sum to 180.

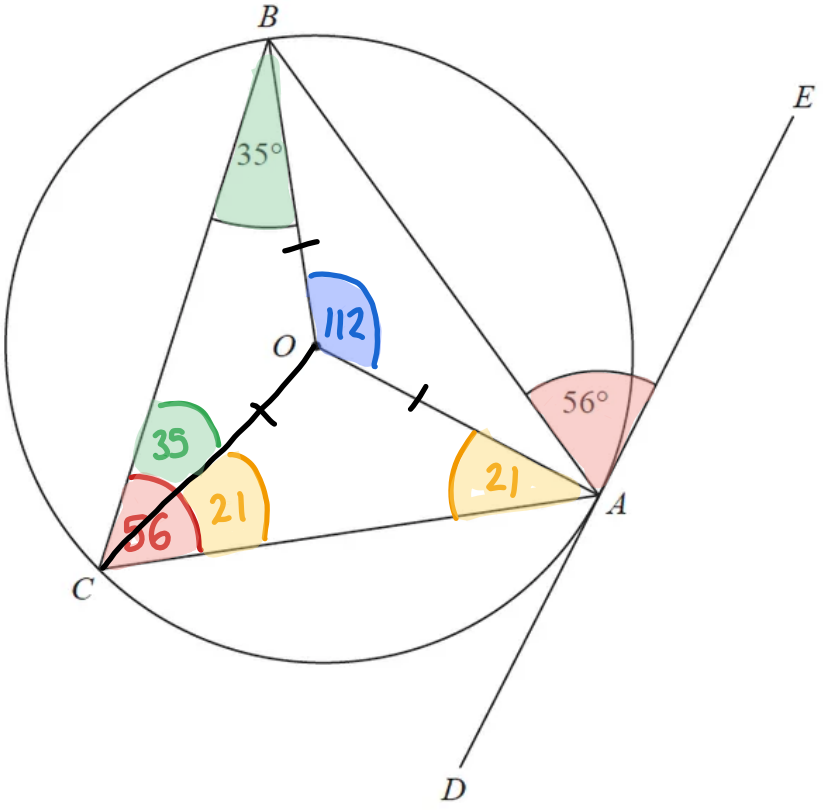
CBD = CDB = 35. Base angles in an isosceles are equal.

ADE = 75. Angles on a straight line sum to 180.

(Total for Question 18 is 5 marks)

[Edexcel, GCSE, Summer 2019, Paper 2H, Q18]

6



A, B and C are points on the circumference of a circle, centre O.
DAE is the tangent to the circle at A.

Angle $BAE = 56°$
Angle $CBO = 35°$

Work out the size of angle CAO.
You must show all your working.

BCA = 56. Angle between a tangent and a chord is the same as the angle in the alternate segment.

OBC = OCB = 35. Base angles in an isosceles are equal.

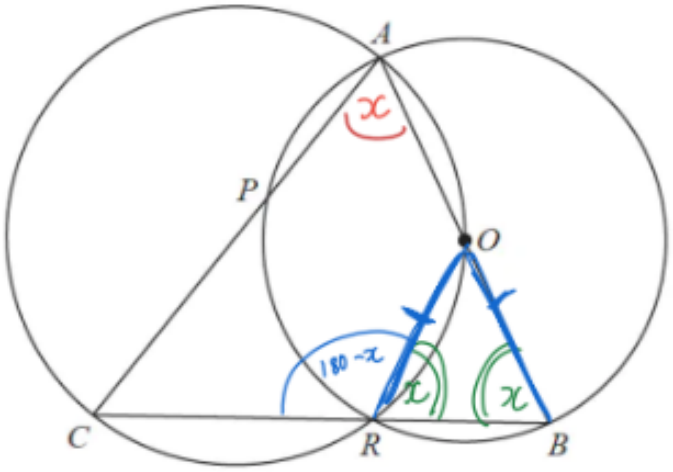
CAO = OCA = 21. Base angles in an isosceles are equal.

21 °

(Total for Question 12 is 3 marks)

[Edexcel, GCSE, Autumn 2018, Paper 1H, Q12]

7



A, B, R and P are four points on a circle with centre O.
A, O, R and C are four points on a different circle.
The two circles intersect at the points A and R.

CPA, CRB and AOB are straight lines.

Prove that angle CAB = angle ABC.

Let $CAB = x$

$CRO = 180 - x$ (opposite angles in a cyclic quadrilateral sum to 180°)

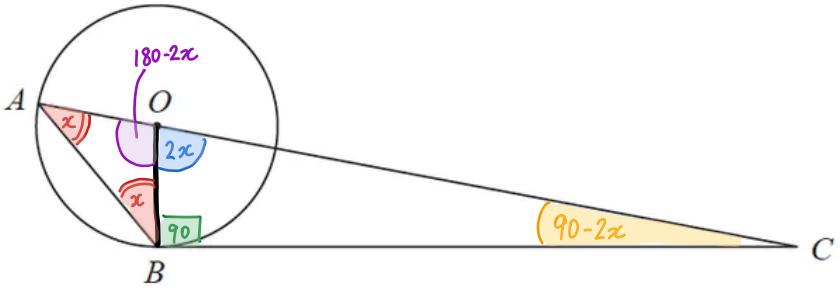
$ORB = x$ (angles on a straight line sum to 180)

$ORB = OBR = x$ (base angles in an isosceles are equal)

(Total for Question 21 is 4 marks)

[Edexcel, GCSE, Autumn 2018, Paper 2H, Q21]

8



A and B are points on a circle, centre O.

BC is a tangent to the circle.
AOC is a straight line.
Angle $ABO = x°$.

Find the size of angle ACB, in terms of x.
Give your answer in its simplest form.
Give reasons for each stage of your working.

ABO = OAB = x. Base angles in an isosceles are equal.

AOB = 180 -2x. Angles in a triangle sum to 180.

BOC = 2x. Angles on a straight line sum to 180.

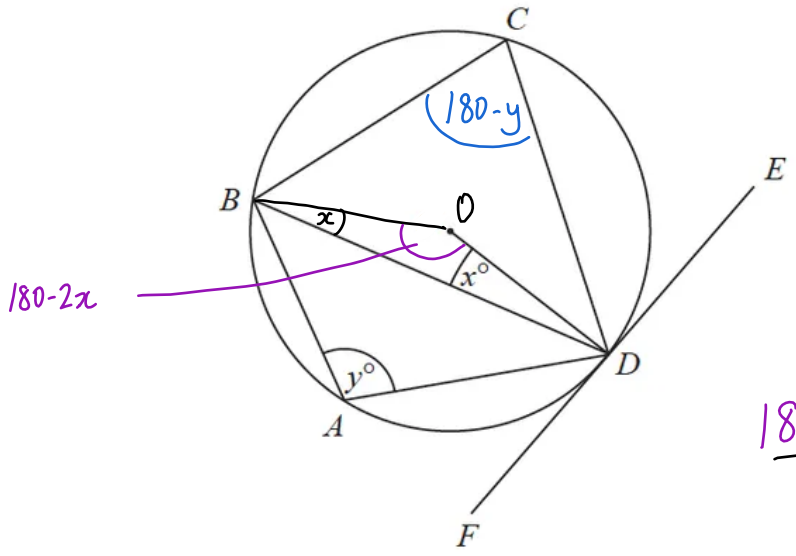
OBC = 90. Angle between a tangent and a radius is 90.

OCB = 90 - 2x. Angles in a triangle sum to 180.

(Total for Question 11 is 5 marks)

[Edexcel, GCSE, Summer 2018, Paper 1H, Q11]

9



$\frac{180-2x}{2} = 180-y$

$90-x = 180-y$

$90+y-x = 90$

$y-x = 90$

A, B, C and D are points on the circumference of a circle, centre O.
FDE is a tangent to the circle.

(a) Show that $y - x = 90$
You must give a reason for each stage of your working.

OBD = ODB = x. Base angles in an isosceles are equal.

BOD = 180 - 2x. Angles in a triangle sum to 180.

BCD = 180 - y. Opposite angles in a cyclical quadrilateral sum to 180.

BOD = 2 x BCD. Angle at the centre is twice the angle at the circumference.

(3)

Dylan was asked to give some possible values for x and y.

He said,

"y could be 200 and x could be 110, because $200 - 110 = 90$"

(b) Is Dylan correct?
You must give a reason for your answer.

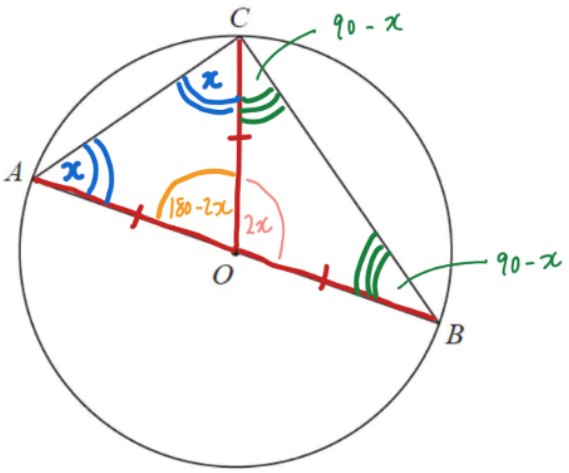
No. Opposite angles in a cyclical quadrilateral sum to 180. So y must be smaller than 180.

(1)

(Total for Question 13 is 4 marks)

[Edexcel, GCSE, Summer 2018, Paper 2H, Q13]

10



A, *B* and *C* are points on the circumference of a circle, centre *O*.
AOB is a diameter of the circle.

Prove that angle *ACB* is 90°
You must **not** use any circle theorems in your proof.

Let angle CAO = x

Angle ACO = x (base angles in an isosceles are equal)

Angle AOC = 180 - 2x (angles in a triangle sum to 180°)

Angle COB = 2x (angles on a straight line sum to 180°)

OBC = OCB = 90 - x

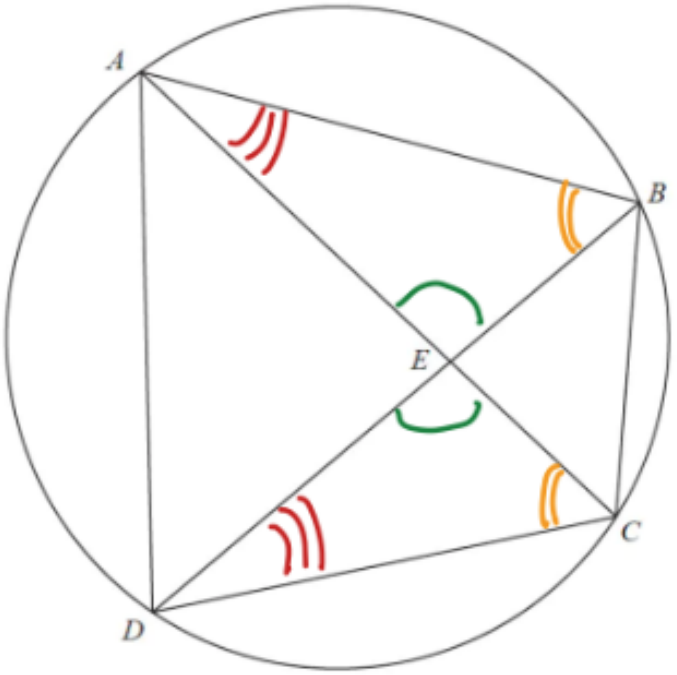
ACB = x + 90 - x = 90°

(Total for Question 20 is 4 marks)

[Edexcel, GCSE, Autumn 2017, Paper 3H, Q20]

11

A, *B*, *C* and *D* are four points on the circumference of a circle.



AEC and *BED* are straight lines.

Prove that triangle *ABE* and triangle *DCE* are similar.
You must give reasons for each stage of your working.

AEB = DEC
vertically opposite angles are equal.

ABE = ACE angles in the same segment are equal.

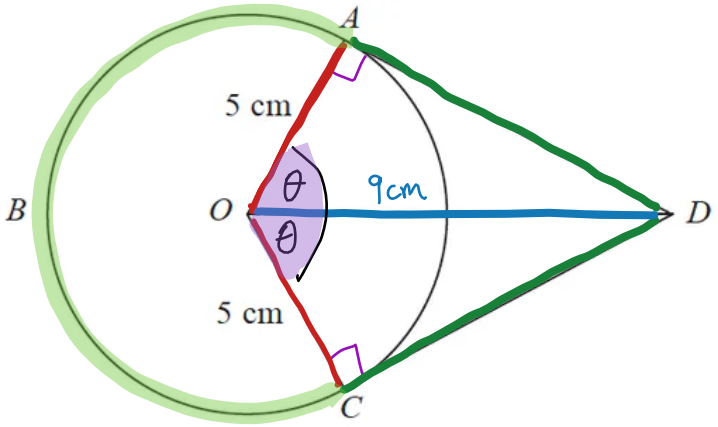
BAE = BDC angles in the same segment are equal.

All angles are equal so the triangles are similar

(Total for Question 15 is 3 marks)

[Edexcel, GCSE, Summer 2017, Paper 2H, Q15]

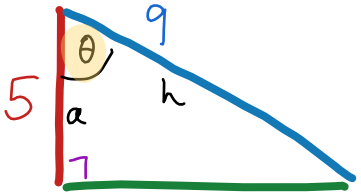
12



A, B and C are points on a circle of radius 5 cm, centre O.
DA and DC are tangents to the circle.
$DO = 9$ cm

C = 10π

Work out the length of arc ABC.
Give your answer correct to 3 significant figures.



$\cos\theta = \frac{a}{h}$

$\theta = \cos^{-1}\left(\frac{5}{9}\right) = 56.25°$

$2\theta = 56.25 \times 2 = 112.5°$

$\text{Arc} = \frac{360 - 112.5}{360} \times 10\pi = 21.598$

21.6 cm

(Total for Question 18 is 5 marks)

[Edexcel, GCSE, Summer 2017, Paper 3H, Q18]