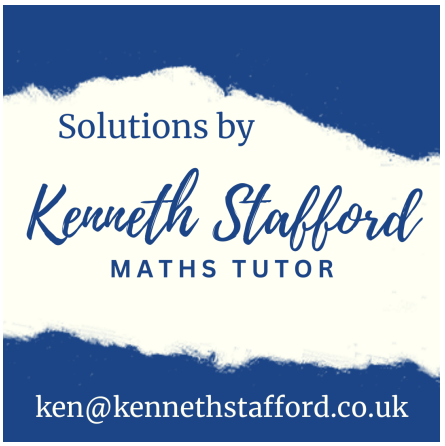
Name: SOLUTIONS

GCSE (1 - 9)
Inverse and Composite Functions





These questions are compiled from *released* Pearson Edexcel GCSE maths exam papers
They contain exam questions from up until the ***November 2021*** exam series
More recent exam papers do exist but are withheld for mock exams
As new exam papers are released these worksheets will gradually be updated
The ***most current*** version of these worksheets will always be available at: kennethstafford.co.uk
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Question	Where the question appeared
1	Edexcel, GCSE, Autumn 2021, Paper 1H, Q21
2	Edexcel, GCSE, Autumn 2020, Paper 1H, Q19
3	Edexcel, GCSE, Autumn 2019, Paper 1H, Q18
4	Edexcel, GCSE, Summer 2019, Paper 1H, Q21
5	Edexcel, GCSE, Autumn 2018, Paper 1H, Q19
6	Edexcel, GCSE, Autumn 2018, Paper 3H, Q10
7	Edexcel, GCSE, Summer 2018, Paper 2H, Q11
8	Edexcel, GCSE, Autumn 2017, Paper 2H, Q22

1

The functions f and g are such that

$$f(x) = 3x^2 + 1 \quad \text{for } x > 0 \qquad \text{and} \qquad g(x) = \frac{4}{x^2} \quad \text{for } x > 0$$

(a) Work out gf(1)

$f(1) = 3(1)^2 + 1 = 4$

$g(4) = \frac{4}{4^2} = \frac{4}{16} = \frac{1}{4}$

$\frac{1}{4}$

(2)

The function h is such that $h = (fg)^{-1}$

(b) Find h(x)

$fg(x) = 3\left(\frac{4}{x^2}\right)^2 + 1$

$= 3\left(\frac{16}{x^4}\right) + 1$

$fg(x) = \frac{48}{x^4} + 1$

$y = \frac{48}{x^4} + 1$

$x = \frac{48}{y^4} + 1$

$x - 1 = \frac{48}{y^4}$

$y^4 = \frac{48}{x-1}$

$y = \sqrt[4]{\frac{48}{x-1}}$

$\sqrt[4]{\frac{48}{x-1}}$

(4)

(Total for Question 21 is 6 marks)

[Edexcel, GCSE, Autumn 2021, Paper 1H, Q21]

2

f and g are functions such that

$$f(x) = \frac{12}{\sqrt{x}} \quad \text{and} \quad g(x) = 3(2x + 1)$$

(a) Find g(5)

$g(5) = 3(2 \times 5 + 1)$

$= 3(11)$

$= 33$

33

(1)

(b) Find gf(9)

$f(9) = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4$

$g(4) = 3(2 \times 4 + 1)$

$= 3(9)$

$= 27$

27

(2)

(c) Find $g^{-1}(6)$

$3(2x + 1) = 6$

$6x + 3 = 6$

$6x = 3$

$x = \frac{3}{6} = \frac{1}{2}$

$\frac{1}{2}$

(2)

(Total for Question 19 is 5 marks)

[Edexcel, GCSE, Autumn 2020, Paper 1H, Q19]

3

The function f is given by

$$f(x) = 2x^3 - 4$$

(a) Show that $f^{-1}(50) = 3$

$y = 2x^3 - 4$

$x = 2y^3 - 4$

$2y^3 = x + 4$

$y^3 = \frac{x+4}{2}$

$y = \sqrt[3]{\frac{x+4}{2}}$

$f^{-1}(x) = \sqrt[3]{\frac{x+4}{2}}$

$f^{-1}(50) = \sqrt[3]{\frac{50+4}{2}}$

$= \sqrt[3]{\frac{54}{2}}$

$= \sqrt[3]{27}$

$= 3$

(2)

The functions g and h are given by

$$g(x) = x + 2 \text{ and } h(x) = x^2$$

(b) Find the values of x for which

$$hg(x) = 3x^2 + x - 1$$

$hg(x) = (x+2)^2 = x^2 + 4x + 4$

$x^2 + 4x + 4 = 3x^2 + x - 1$

$3x^2 + x - 1 = x^2 + 4x + 4$

$2x^2 - 3x - 5 = 0$

$(2x - 5)(x + 1) = 0$

$x = \frac{5}{2}$ $x = -1$

2.5 and -1

(4)

(Total for Question 18 is 6 marks)

[Edexcel, GCSE, Autumn 2019, Paper 1H, Q18]

4

The functions f and g are such that

$$f(x) = 3x - 1 \quad \text{and} \quad g(x) = x^2 + 4$$

(a) Find $f^{-1}(x)$

$y = 3x - 1$

$x = 3y - 1$

$x + 1 = 3y$

$y = \frac{x+1}{3}$

$f^{-1}(x) =$ $\frac{x+1}{3}$

(2)

Given that $fg(x) = 2gf(x)$,

(b) show that $15x^2 - 12x - 1 = 0$

$fg(x) = 3(x^2+4) - 1 = 3x^2 + 11$

$gf(x) = (3x-1)^2 + 4 = 9x^2 - 6x + 5$

$3x^2 + 11 = 2(9x^2 - 6x + 5)$

$3x^2 + 11 = 18x^2 - 12x + 10$

$15x^2 - 12x - 1 = 0$

(5)

(Total for Question 21 is 7 marks)

[Edexcel, GCSE, Summer 2019, Paper 1H, Q21]

5

For all values of x

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

x^2+2x+1 $2x-2$

(a) Show that $gf(x) = 2x(x + 2)$

$gf(x) = 2(x^2+2x+1)-2$

$gf(x) = 2x^2+4x = 2x(x+2)$

(2)

(b) Find $g^{-1}(7)$

$2x-2=7$

$2x=9$

$x=4.5$

4.5

(2)

(Total for Question 19 is 4 marks)

[Edexcel, GCSE, Autumn 2018, Paper 1H, Q19]

6

$f(x) = 4\sin x°$

(a) Find f(23)
Give your answer correct to 3 significant figures.

$4\sin(23) = 1.5629$

1.56

(1)

$g(x) = 2x - 3$

(b) Find fg(34)
Give your answer correct to 3 significant figures.

$g(34) = 2(34) - 3 = 65$

$f(65) = 4\sin(65)$

$= 3.6252$

3.63

(2)

$h(x) = (x + 4)^2$

Ivan needs to solve the following equation $h(x) = 25$

He writes

$(x + 4)^2 = 25$
$x + 4 = 5$
$x = 1$

$\sqrt{25} = 5$ or -5

This is not fully correct.

The other solution is -9

(c) Explain why.

$x + 4 = -5$

$x = -9$

(1)

(Total for Question 10 is 4 marks)

[Edexcel, GCSE, Autumn 2018, Paper 3H, Q10]

7

f and g are functions such that

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$

(a) Find f(–5)

$\frac{2}{(-5)^2} = \frac{2}{25}$

$\frac{2}{25}$

.....

(1)

(b) Find fg(1)

$g(1) = 4(1)^3 = 4$

$g(4) = \frac{2}{(4)^2} = \frac{2}{16} = \frac{1}{8}$

.....

(2)

(Total for Question 11 is 3 marks)

[Edexcel, GCSE, Summer 2018, Paper 2H, Q11]

8

The functions f and g are such that

$$f(x) = 5x + 3 \qquad g(x) = ax + b \qquad \text{where } a \text{ and } b \text{ are constants.}$$

$g(3) = 20$ and $f^{-1}(33) = g(1)$

Find the value of a and the value of b.

$g(3) = 20$

$3a + b = 20$

$5x + 3 = 33$

$5x = 30$

$x = 6$

$g(1) = 6$

$1a + b = 6$

$$\begin{array}{r} 3a + b = 20 \\ -(a + b = 6) \\ \hline 2a = 14 \end{array}$$

$a = 7$

$7 + b = 6$

$b = -1$

$a =$ 7

$b =$ -1

(Total for Question 22 is 5 marks)

[Edexcel, GCSE, Autumn 2017, Paper 2H, Q22]