Name: SOLUTIONS

GCSE (1 - 9)

Iteration

These questions represent all of the times that this topic has appeared in unlocked Pearson Edexcel GCSE maths papers.

Question	Where the question appeared
1	Edexcel, GCSE, Summer 2017, Paper 3H, Q07
2	Edexcel, GCSE, Autumn 2017, Paper 2H, Q18
3	Edexcel, GCSE, Autumn 2017, Paper 3H, Q15
4	Edexcel, GCSE, Summer 2018, Paper 3H, Q18
5	Edexcel, GCSE, Autumn 2018, Paper 3H, Q13
6	Edexcel, GCSE, Autumn 2019, Paper 2H, Q22
7	Edexcel, GCSE, Autumn 2020, Paper 2H, Q19
8	N8
9	N9

1

Using $x_{n+1} = -2 - \frac{4}{x_n^2}$

with $x_0 = -2.5$

(a) find the values of x_1, x_2 and x_3

$x_0 = -2.5$

$x_1 = -2 - \frac{4}{(-2.5)^2} = -2.64$

$x_2 = -2 - \frac{4}{(-2.64)^2} = -2.57392$

$x_3 = -2 - \frac{4}{(-2.57)^2} = -2.603767$

$x_1 =$ -2.64

$x_2 =$ -2.57392

$x_3 =$ -2.603767

(3)

(b) Explain the relationship between the values of x_1, x_2 and x_3 and the equation $x^3 + 2x^2 + 4 = 0$

Connection between equation and iterative form in (a) e.g. rearrangement

Statement e.g. iteration is an estimation of a solution `1

(2)

(Total for Question 16 is 5 marks)

[Edexcel, GCSE, Summer 2017, Paper 3H, Q07]

2

At time $t = 0$ hours a tank is full of water.

Water leaks from the tank.
At the end of every hour there is 2% less water in the tank than at the start of the hour.

The volume of water, in litres, in the tank at time t hours is V_t

Given that

$$V_0 = 2000$$
$$V_{t+1} = kV_t$$

write down the value of k.

100% - 2% = 98%

98% = 0.98

$k =$ 0.98

(Total for Question 18 is 1 mark)

[Edexcel, GCSE, Autumn 2017, Paper 2H, Q18]

3

(a) Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$

$x = 0$ $\quad (0)^3 + 7(0) - 5 = -5$

$x = 1$ $\quad (1)^3 + 7(1) - 5 = 3$

as there is a sign change there must be at least one root between x = 0 and x = 1

(2)

(b) Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \frac{5}{x^2 + 7}$

$x^3 + 7x - 5 = 0$

$x^3 + 7x = 5$

$x(x^2 + 7) = 5$

$x = \frac{5}{x^2 + 7}$

(2)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \frac{5}{x_n^2 + 7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$

$x_0 = 1$

$x_1 = \frac{5}{(\,)^2 + 7} = 0.625$

$x_2 = \frac{5}{(0.625)^2 + 7} = 0.67653$

$x_3 = \frac{5}{(0.67653)^2 + 7} = 0.670448$

0.670448

(3)

(d) By substituting your answer to part (c) into $x^3 + 7x - 5$, comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$

$(0.670448)^3 + 7(0.6704) - 5 = 0.00549$

This is close to zero so our estimate is pretty accurate.

(2)

(Total for Question 15 is 9 marks)

[Edexcel, GCSE, Autumn 2017, Paper 3H, Q15]

4

(a) Show that the equation $x^3 + x = 7$ has a solution between 1 and 2

$x = 1$ $\quad (1)^3 + (1) = 2$

$x = 2$ $\quad (2)^3 + (2) = 10$

x=1 and x=2 give us values above and below seven so the solution must be between 1 and 2.

(2)

(b) Show that the equation $x^3 + x = 7$ can be rearranged to give $x = \sqrt[3]{7 - x}$

$x^3 + x = 7$

$x^3 = 7 - x$

$x = \sqrt[3]{7 - x}$

(1)

(c) Starting with $x_0 = 2$, use the iteration formula $x_{n+1} = \sqrt[3]{7 - x_n}$ three times to find an estimate for a solution of $x^3 + x = 7$

$x_0 = 2$

$x_1 = \sqrt[3]{7 - 2} = 1.70997$

$x_2 = \sqrt[3]{7 - 1.70997} = 1.74241$

$x_3 = \sqrt[3]{7 - 1.74241} = 1.73884$

1.73884

(3)

(Total for Question 18 is 6 marks)

[Edexcel, GCSE, Summer 2018, Paper 3H, Q18]

5

The number of animals in a population at the start of year t is P_t
The number of animals at the start of year 1 is 400

Given that

$$P_{t+1} = 1.01P_t$$

work out the number of animals at the start of year 3

$P_1 = 400$

$P_2 = 1.01 \times 400 = 404$

$P_3 = 1.01 \times 404 = 408.04$

408

(Total for Question 13 is 2 marks)

[Edexcel, GCSE, Autumn 2018, Paper 3H, Q13]

6

The number of rabbits on a farm at the end of month n is P_n
The number of rabbits at the end of the next month is given by $P_{n+1} = 1.2P_n - 50$

At the end of March there are 200 rabbits on the farm.

(a) Work out how many rabbits there will be on the farm at the end of June.

March = 200

April = 1·2 x 200 - 50 = 190

May = 1·2 x 190 - 50 = 178

June = 1·2 x 178 - 50 = 163·6

either answer

163 or 164

(3)

(b) Considering your results in part (a), suggest what will happen to the number of rabbits on the farm after a long time.

There will eventually be no rabbits left

(1)

(Total for Question 22 is 4 marks)

[Edexcel, GCSE, Autumn 2019, Paper 2H, Q22]

7

A hot air balloon is descending.
The height of the balloon n minutes after it starts to descend is h_n metres.

The height of the balloon $(n+1)$ minutes after it starts to descend, h_{n+1} metres, is given by

$$h_{n+1} = K \times h_n + 20 \quad \text{where } K \text{ is a constant.}$$

The balloon starts to descend from a height of 1200 metres at 09 15
At 09 16 the height of the balloon is 1040 metres.

Work out the height of the balloon at 09 18

0915 = 1200m

0916 = 1040m

$1040 = K \times 1200 + 20$

$1020 = 1200K$

$K = \frac{1020}{1200} = 0.85$

0917 = $0.85 \times 1040 + 20 = 904$

0918 = $0.85 \times 904 + 20 = 788.4$

788.4 ..... m

(Total for Question 19 is 4 marks)

[Edexcel, GCSE, Autumn 2020, Paper 2H, Q19]