Name: SOLUTIONS

GCSE (1 - 9) Proof	Solutions by Kenneth Stafford MATHS TUTOR ken@kennethstafford.co.uk

These questions are compiled from released Pearson Edexcel GCSE maths exam papers

They contain exam questions from up until the ***November 2023*** exam series

More recent exam papers do exist but are withheld for mock exams

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Past Papers:	**Similar Worksheets:**
Solutions:	**This worksheet:**

1

n is an integer greater than 1

Prove algebraically that $n^2 - 2 - (n-2)^2$ is always an even number.

$n^2 - 2 - (n-2)^2$

$n^2 - 2 - (n^2 - 4n + 4)$

$n^2 - 6 - (n^2 - 4n)$

$n^2 + 4n - 6 - (n^2)$

$4n - 6$

$= 2(2n-3)$ which is a multiple of 2 and therefore even.

(Total for Question 16 is 4 marks)

[Edexcel, GCSE, Summer 2017, Paper 1H, Q16]

2

n is an integer.

Prove algebraically that the sum of $\frac{1}{2}n(n+1)$ and $\frac{1}{2}(n+1)(n+2)$ is always a square number.

$\frac{1}{2}n(n+1) = \frac{1}{2}n^2 + \frac{1}{2}n$

$\frac{1}{2}(n+1)(n+2) = \frac{1}{2}(n^2+3n+2) = \frac{1}{2}n^2 + 1\frac{1}{2}n + 1$

$\frac{1}{2}n^2 + \frac{1}{2}n$

$+ \frac{1}{2}n^2 + 1\frac{1}{2}n + 1$

$n^2 + 2n + 1 = (n+1)(n+1)$

which is a square number.

(Total for Question 17 is 2 marks)

[Edexcel, GCSE, Autumn 2017, Paper 1H, Q17]

3

Prove that the square of an odd number is always 1 more than a multiple of 4

$2n+1$ is odd

$(2n+1)(2n+1) = 4n^2+4n+1$

$= 4(n^2+n)+1$

$4(n^2+n)$ is a multiple of 4.

$4(n^2+n)+1$ is 1 more than a multiple of 4.

(Total for Question 12 is 4 marks)

[Edexcel, GCSE, Summer 2018, Paper 1H, Q12]

4

Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

$2n+1$ and $2n+3$ are consecutive odd numbers.

$(2n+3)(2n+3) = 4n^2 + 12n + 9$

$(2n+1)(2n+1) = 4n^2 + 4n + 1$

$8n + 8 = 8(n+1)$

which is a multiple of 8.

(Total for Question 15 is 3 marks)

[Edexcel, GCSE, Autumn 2018, Paper 3H, Q15]

5

Given that n can be any integer such that $n > 1$, prove that $n^2 - n$ is never an odd number.

$n^2 - n = n(n-1)$

n and $n-1$ are consecutive numbers.

Therefore one is odd and one is even.

odd x even = even.

So $n^2 - n$ is never odd.

(Total for Question 13 is 2 marks)

[Edexcel, GCSE, Summer 2019, Paper 1H, Q13]

6

Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4

$2n$ and $2n+2$ are consecutive even numbers.

$(2n)(2n) = 4n^2$

$(2n+2)(2n+2) = 4n^2 + 4n + 4$

$+$

$8n^2 + 4n + 4 = 4(2n^2 + n + 1)$

Which is a multiple of 4

(Total for Question 15 is 3 marks)

[Edexcel, GCSE, Autumn 2019, Paper 3H, Q15]

7

An expression for the nth term of the sequence of triangular numbers is $\frac{n(n+1)}{2}$

Prove that the sum of any two consecutive triangular numbers is a square number.

$$\frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2}$$

$$\frac{n^2+n}{2} + \frac{n^2+3n+2}{2} = \frac{2n^2+4n+2}{2}$$

$$= n^2 + 2n + 1 = (n+1)(n+1)$$

$$= (n+1)^2$$

Square number

(Total for Question 13 is 3 marks)

[Edexcel, GCSE, Summer 2023, Paper 3H, Q13]

8

(a) Prove that

$$(2m+1)^2-(2n-1)^2=4(m+n)(m-n+1)$$

$4m^2+4m+1-[4n^2-4n+1]=4(m^2-mn+m+nm-n^2+n)$

$4m^2-4n^2+4m+4n=4(m^2-n^2+m+n)$

$4(m^2-n^2+m+n)=4(m^2-n^2+m+n)$

(3)

Sophia says that the result in part (a) shows that the difference of the squares of any two odd numbers must be a multiple of 4

(b) Is Sophia correct?
You must give reasons for your answer.

Yes. $(2m+1)^2-(2m-1)^2$ is the difference of the squares of 2 odd numbers and is a multiple of 4.

(1)

(Total for Question 16 is 4 marks)

[Edexcel, GCSE, November 2022, Paper 1H, Q16]