Name: SOLUTIONS

GCSE (1 - 9) Vector Proof	Solutions by Kenneth Stafford MATHS TUTOR ken@kennethstafford.co.uk

These questions are compiled from released Pearson Edexcel GCSE maths exam papers

They contain exam questions from up until the ***November 2023*** exam series

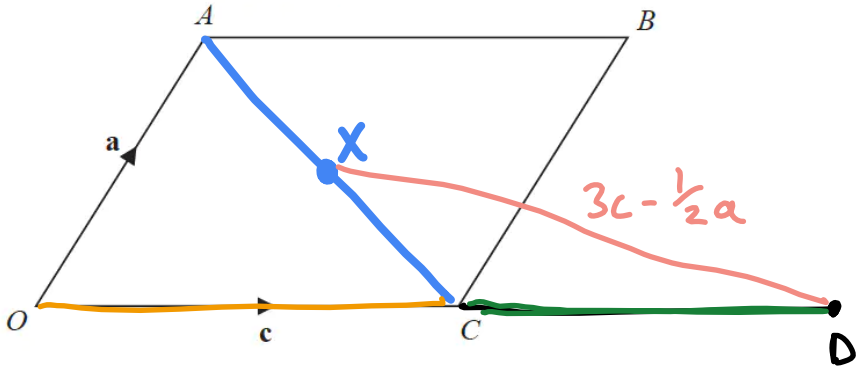
More recent exam papers do exist but are withheld for mock exams

As new exam papers are released these worksheets will gradually be updated

The **most current** version of these worksheets will always be available at: kennethstafford.co.uk

Past Papers:	Similar Worksheets:
Solutions:	This worksheet:

1



OABC is a parallelogram.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

$\overrightarrow{AC} = c - a$

$\overrightarrow{XC} = \frac{1}{2}c - \frac{1}{2}a$

X is the midpoint of the line *AC*.
OCD is a straight line so that $OC : CD = k : 1$

Given that $\overrightarrow{XD} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$

find the value of *k*.

$\overrightarrow{XC} + \overrightarrow{CD} = \overrightarrow{XD}$

$\frac{1}{2}c - \frac{1}{2}a + \overrightarrow{CD} = 3c - \frac{1}{2}a$

$\overrightarrow{CD} = 2.5c$

$OC : CD$

$c : 2.5c$ (÷2.5)

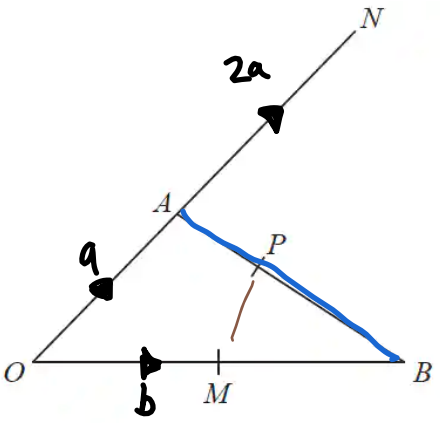
$\frac{2}{5}c : 1c$ (÷2.5)

$k =$ $\frac{2}{5}$

(Total for Question 19 is 4 marks)

[Edexcel, GCSE, Summer 2017, Paper 1H, Q19]

2



OAN, *OMB* and *APB* are straight lines.
$AN = 2OA$.
M is the midpoint of OB.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

$\overrightarrow{AP} = k\overrightarrow{AB}$ where k is a scalar quantity.

Given that MPN is a straight line, find the value of k.

$\overrightarrow{AB} = b - a$

$\overrightarrow{MN} = \overrightarrow{MO} + \overrightarrow{ON}$

$\overrightarrow{MN} = -\frac{1}{2}b + 3a$

$\overrightarrow{AP} = k(b-a)$

$\overrightarrow{MP} = \frac{1}{2}b + a + k(b-a) = (1-k)a + (k-\frac{1}{2})b$

$\overrightarrow{MN} = x\overrightarrow{MP}$ $\quad 3a - \frac{1}{2}b = x[(1-k)a + (k-\frac{1}{2})b]$

a) $3 = x(1-k)$

$x = \frac{3}{1-k}$

b) $-\frac{1}{2} = x(k-\frac{1}{2})$

$x = \frac{-1}{2(k-\frac{1}{2})}$

$\frac{3}{1-k} = \frac{-1}{2(k-\frac{1}{2})}$

$6k - 3 = k - 1$

$5k = 2$

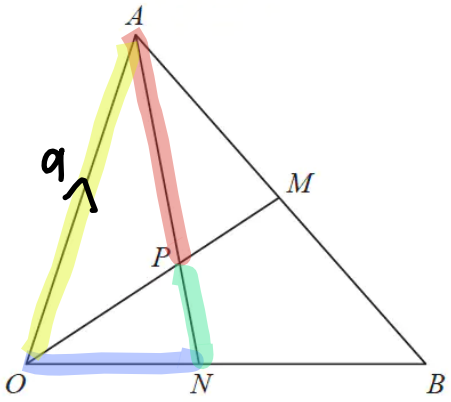
$k = \frac{2}{5}$

$k = \frac{2}{5}$

(Total for Question 21 is 5 marks)

[Edexcel, GCSE, Autumn 2017, Paper 3H, Q21]

3



$\overrightarrow{AB} = -a + b$

$\overrightarrow{AM} = -0.5a + 0.5b$

$\overrightarrow{OM} = 0.5a + 0.5b$

$\overrightarrow{OP} = 0.3a + 0.3b$

$\overrightarrow{PM} = 0.2a + 0.2b$

OAB is a triangle.
OPM and *APN* are straight lines.
M is the midpoint of *AB*.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

OP : *PM* = 3 : 2

Work out the ratio *ON* : *NB*

$\overrightarrow{AP} = \overrightarrow{AO} + \overrightarrow{OP} = -a + 0.3a + 0.3b = -0.7a + 0.3b$

$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AP} + \overrightarrow{PN}$

$\overrightarrow{ON} = a + (-0.7a + 0.3b) + (-0.3a + \frac{9}{70}b)$

$\overrightarrow{ON} = \frac{3}{7}b$

ON : NB

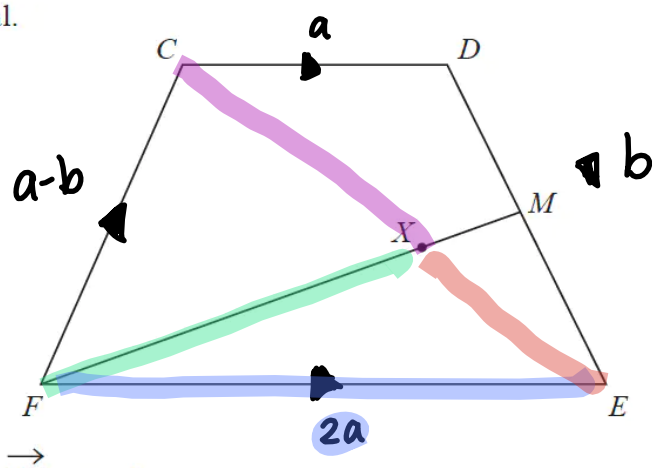
3 : 4

(Total for Question 21 is 5 marks)

[Edexcel, GCSE, Autumn 2018, Paper 1H, Q21]

4

CDEF is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{FE}$ in terms of **a** and/or **b**.
Give your answer in its simplest form.

$\overrightarrow{FE} = \overrightarrow{FC} + \overrightarrow{CD} + \overrightarrow{DE} = a-b+a+b$

$= 2a$

(2)

M is the midpoint of *DE*.
X is the point on *FM* such that $FX : XM = n : 1$
CXE is a straight line.

(b) Work out the value of *n*.

$\overrightarrow{CE} = a+b$

$\overrightarrow{CX} = k(a+b)$

$\overrightarrow{FM} = 2a - 0.5b$

$\overrightarrow{FX} = a-b+k(a+b)$

$\overrightarrow{FE} = \overrightarrow{FX} + \overrightarrow{XE}$

$2a = k(2a-0.5b) + y(a+b)$

$2a = 0.8(2a-0.5b) + 0.4(a+b)$

$k = 0.8$

$FX : XM$

$0.8 : 0.2$

$4 : 1$

$n =$ 4

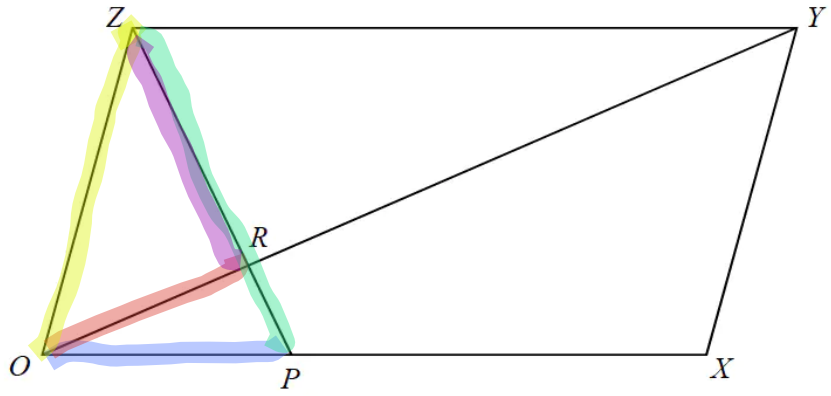
(4)

(Total for Question 20 is 6 marks)

[Edexcel, GCSE, Summer 2019, Paper 2H, Q20]

5

OXYZ is a parallelogram.



$\overrightarrow{OX} = \mathbf{a}$

$\overrightarrow{OY} = \mathbf{b}$

P is the point on OX such that $OP : PX = 1 : 2$
R is the point on OY such that $OR : RY = 1 : 3$

Work out, in its simplest form, the ratio $ZP : ZR$
You must show all your working.

$\overrightarrow{OP} = \frac{1}{3}a$ $\quad \overrightarrow{PX} = \frac{2}{3}a$

$\overrightarrow{OR} = \frac{1}{4}b$ $\quad \overrightarrow{RY} = \frac{3}{4}b$

$\overrightarrow{OZ} = b - a$

$\overrightarrow{ZO} = a - b$

$\overrightarrow{ZP} = \overrightarrow{ZO} + \overrightarrow{OP} = a - b + \frac{1}{3}a = \frac{4}{3}a - b$

$\overrightarrow{ZR} = \overrightarrow{ZO} + \overrightarrow{OR} = a - b + \frac{1}{4}b = a - \frac{3}{4}b$

$\overrightarrow{ZP} = \frac{4}{3}a - b = 4\left(\frac{1}{3}a - \frac{1}{4}b\right)$

$\overrightarrow{ZR} = a - \frac{3}{4}b = 3\left(\frac{1}{3}a - \frac{1}{4}b\right)$

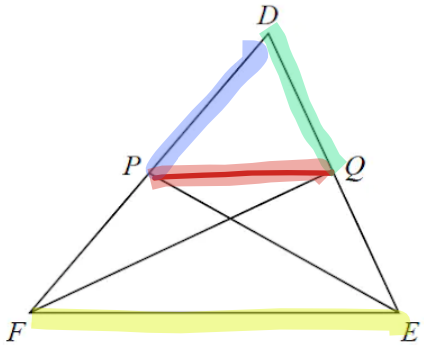
4 : 3

(Total for Question 24 is 5 marks)

[Edexcel, GCSE, Autumn 2019, Paper 3H, Q24]

6

DEF is a triangle.



$\overrightarrow{FP} = \frac{1}{2}a = \overrightarrow{PD}$

$\overrightarrow{DE} = -a + b$

$\overrightarrow{DQ} = -\frac{1}{2}a + \frac{1}{2}b$

P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

$\overrightarrow{PQ} = \overrightarrow{PD} + \overrightarrow{DQ}$

$\overrightarrow{PQ} = \frac{1}{2}a + -\frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{PQ} = \frac{1}{2}b$

$\overrightarrow{FE} = 2(\overrightarrow{PQ})$

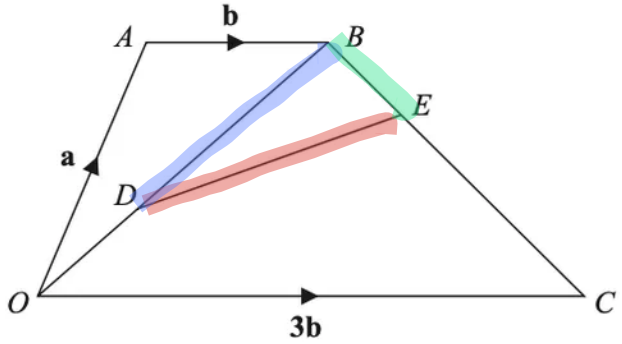
Therefore parallel

(Total for Question 21 is 4 marks)

[Edexcel, GCSE, Autumn 2020, Paper 1H, Q21]

7

OABC is a trapezium.



$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{AB} = \mathbf{b}$
$\overrightarrow{OC} = \mathbf{3b}$

D is the point on *OB* such that $OD : DB = 2 : 3$
E is the point on *BC* such that $BE : EC = 1 : 4$

Work out the vector $\overrightarrow{DE}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{OB} = a + b$

$\overrightarrow{OD} = \frac{2}{5}a + \frac{2}{5}b$

$\overrightarrow{DB} = \frac{3}{5}a + \frac{3}{5}b$

$\overrightarrow{BC} = -b - a + 3b = 2b - a$

$\overrightarrow{BE} = \frac{1}{5}(2b - a) = \frac{2}{5}b - \frac{1}{5}a$

$\overrightarrow{DE} = \overrightarrow{DB} + \overrightarrow{BE}$

$\overrightarrow{DE} = \frac{3}{5}a + \frac{3}{5}b + \frac{2}{5}b - \frac{1}{5}a$

$\overrightarrow{DE} = \frac{2}{5}a + b$

.....

(Total for Question 18 is 4 marks)

[Edexcel, GCSE, Autumn 2020, Paper 1H, Q21]

8

A, B and C are three points such that

$\overrightarrow{AB} = 3\mathbf{a} + 4\mathbf{b}$

$\overrightarrow{AC} = 15\mathbf{a} + 20\mathbf{b}$

(a) Prove that A, B and C lie on a straight line.

(2)

D, E and F are three points on a straight line such that

$\overrightarrow{DE} = 3\mathbf{e} + 6\mathbf{f}$

$\overrightarrow{EF} = -10.5\mathbf{e} - 21\mathbf{f}$

(b) Find the ratio

length of DF : length of DE

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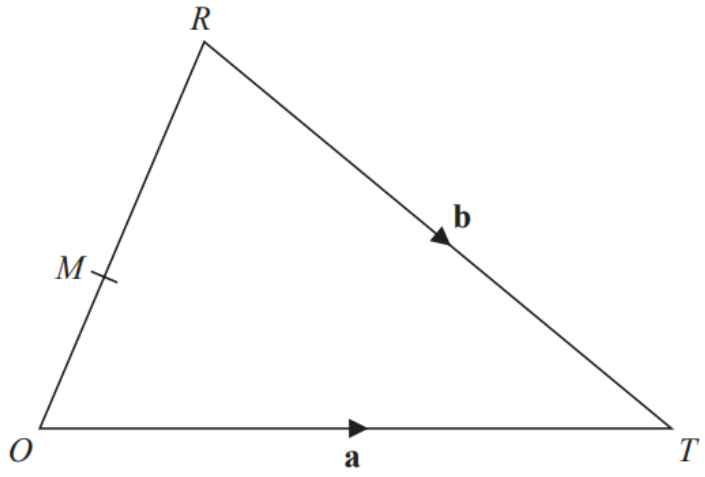
(3)

(Total for Question 15 is 5 marks)

[Edexcel, GCSE, Summer 2022, Paper 1H, Q15]

9

ORT is a triangle.



$\overrightarrow{OT} = \mathbf{a}$ $\overrightarrow{RT} = \mathbf{b}$

M is the point on OR such that $OM : MR = 2 : 3$

Express $\overrightarrow{MT}$ in terms of **a** and **b**.
Give your answer in its simplest form.

.....

(Total for Question 20 is 4 marks)

[Edexcel, GCSE, Summer 2023, Paper 2H, Q20]

10

Given that the vector $a\begin{pmatrix}2\\6\end{pmatrix}+b\begin{pmatrix}8\\2\end{pmatrix}$ is parallel to the vector $\begin{pmatrix}13\\6\end{pmatrix}$

find an expression for b in terms of a.

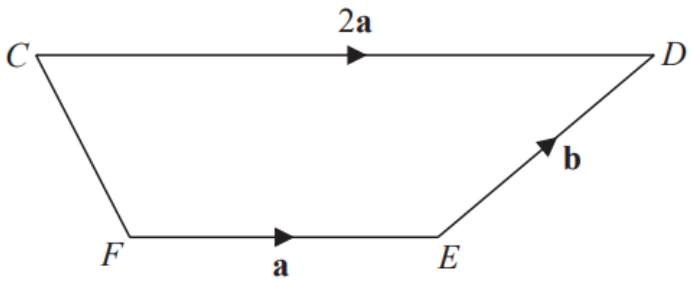
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(Total for Question 22 is 3 marks)

[Edexcel, GCSE, Summer 2023, Paper 3H, Q22]

11

CDEF is a quadrilateral.



$\overrightarrow{FE} = \mathbf{a}$ $\overrightarrow{ED} = \mathbf{b}$ $\overrightarrow{CD} = 2\mathbf{a}$

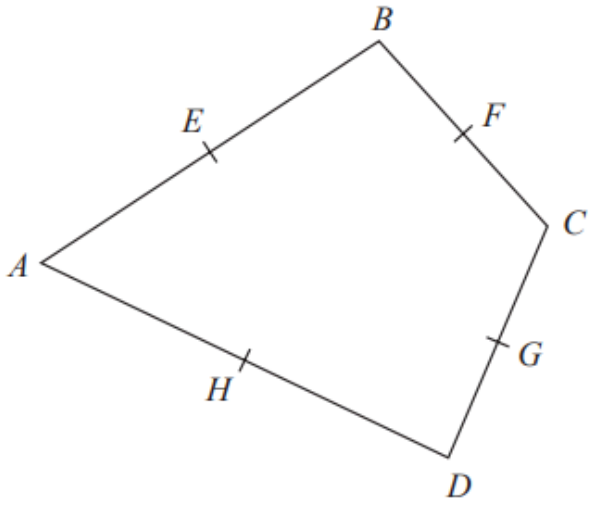
The point *P* is such that *CEP* is a straight line and that *CE* = *EP*

Use a vector method to prove that *CF* is parallel to *DP*.

(Total for Question 24 is 4 marks)

[Edexcel, GCSE, November 2022, Paper 3H, Q24]

12



$ABCD$ is a quadrilateral.

E, F, G and H are the midpoints of AB, BC, CD and DA.

$$\overrightarrow{AH} = \mathbf{a} \qquad \overrightarrow{AE} = \mathbf{b} \qquad \overrightarrow{DG} = \mathbf{c}$$

Prove, using vectors, that $EFGH$ is a parallelogram.

(Total for Question 18 is 4 marks)

[Edexcel, GCSE, November 2023, Paper 2H, Q18]